Beams, Shear Force & Bending Moment Diagrams

- Shall approach this through examining BEAMs.
- There are a number of steps that you MUST go through to get the solution.
- Make sure you follow this every time – makes solution a lot easier!

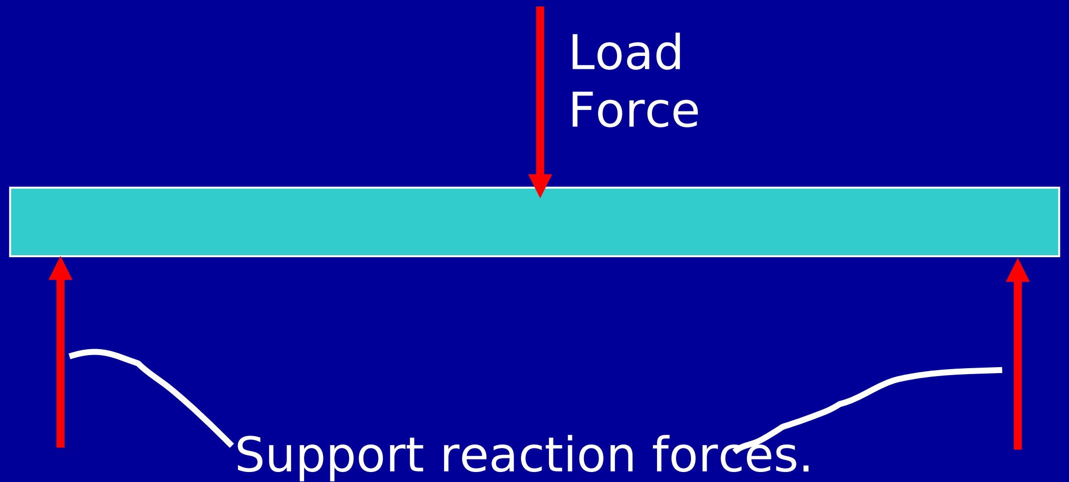
Bending of Beams

Many objects in everyday life can be analysed as beams.

The design and analysis of any structural member requires knowledge of the internal loadings acting within it, not only when it is in place and subjected to service loads, but also when it is being hoisted.

In this lecture we will discuss how engineers determine these loadings.

If we ignore **mass of the beam**, the **forces on the beam** are as shown:

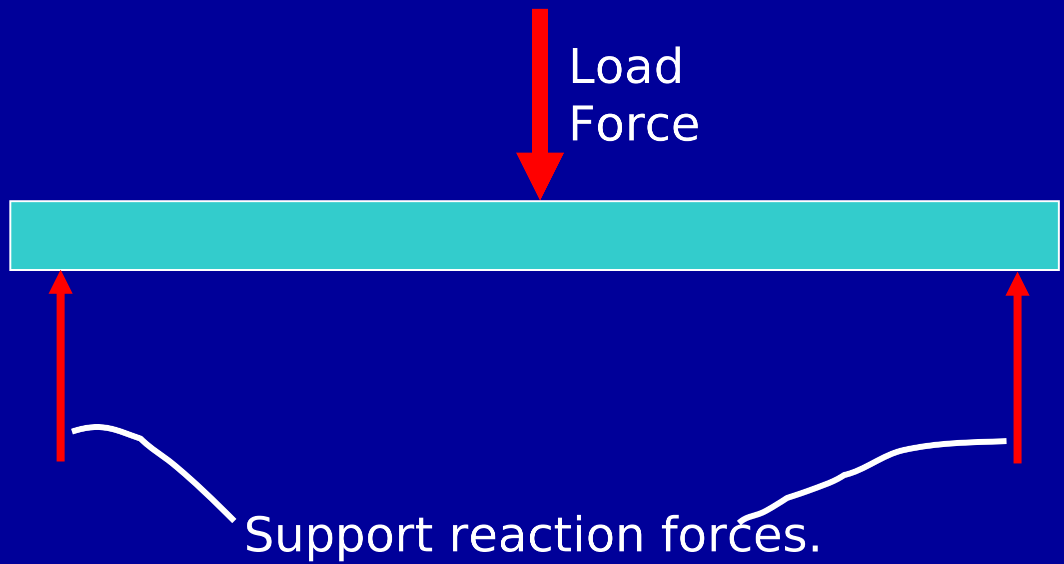


We shall normally ignore the beam mass in these lectures.

Bending of Beams

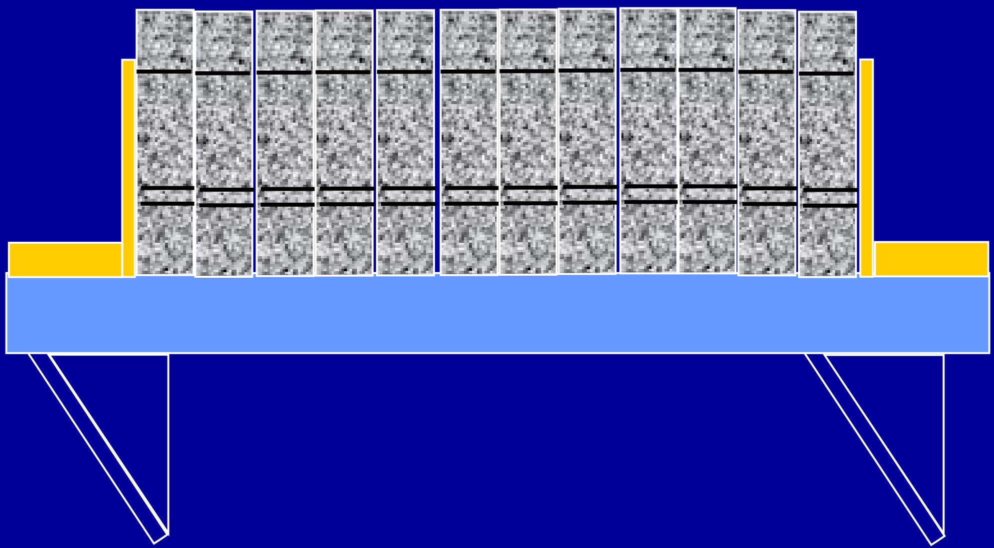
In the figure, the force shown downwards is acting on the beam.

It is a **point load**, acting at a **single point** on the beam.

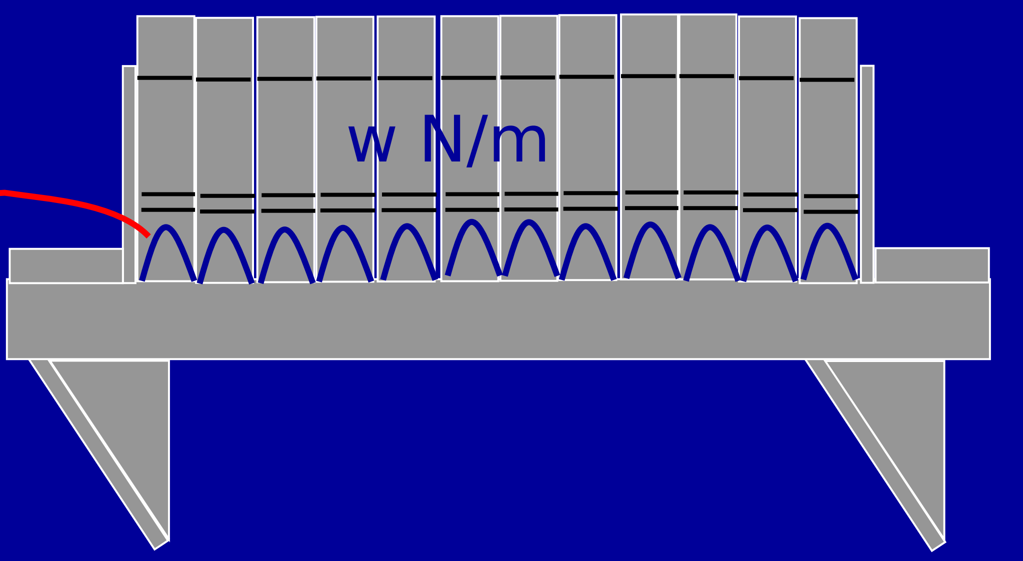


Bending of Beams

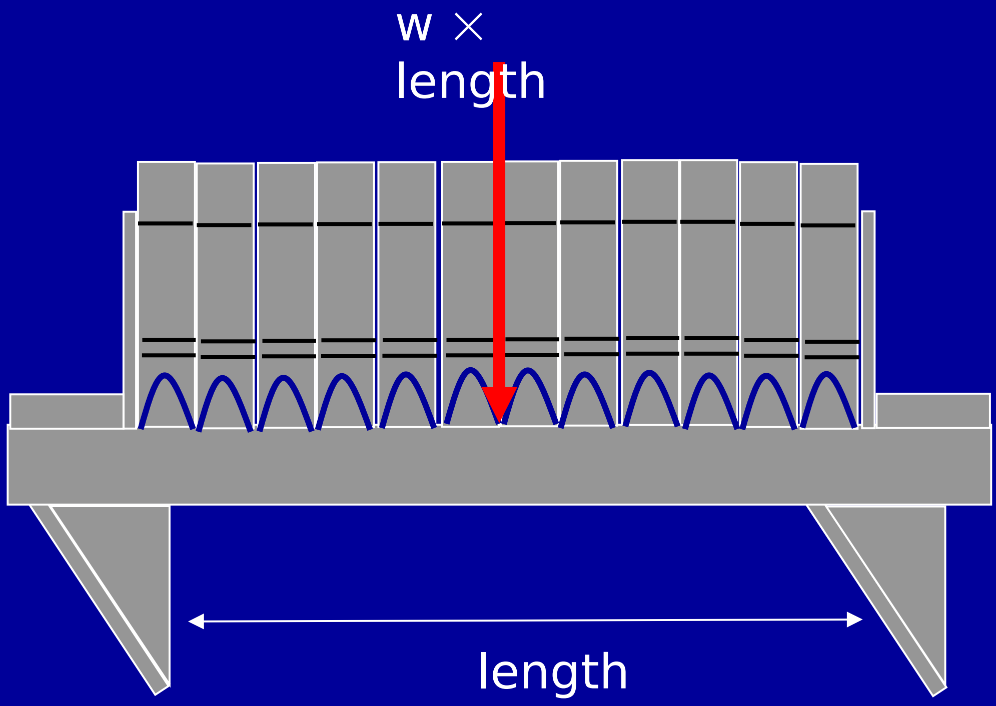
However, in the figure below, the books are exerting a uniformly distributed load (UDL) on the shelf.



A UDL is represented as shown in figure:



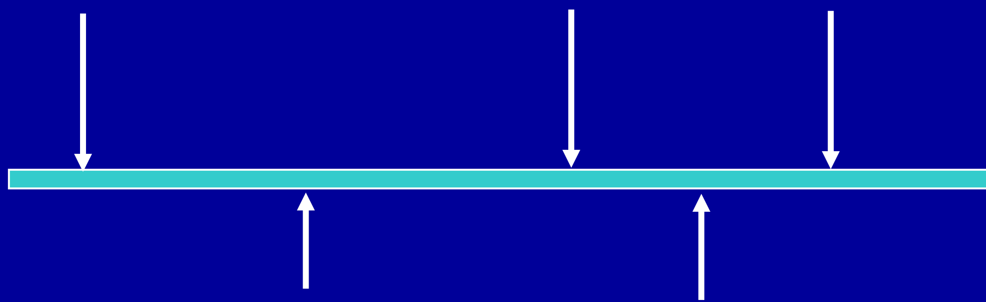
Total load is = w × length



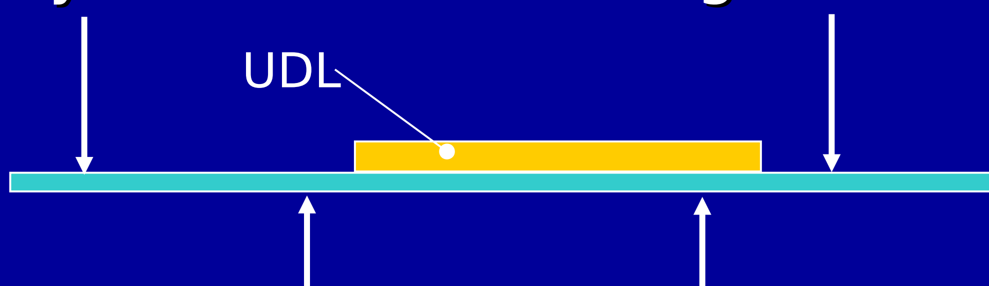
In calculating Reactions the UDL is considered to **act at the mid-point** of its length.

Beam loading:

Point loading

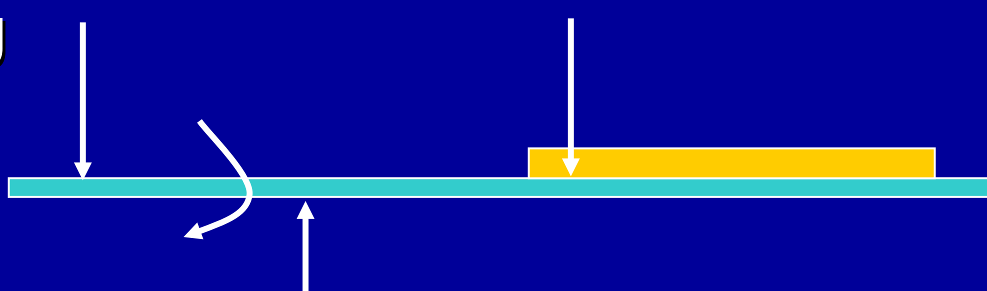


Uniformly Distributed Loading (UDL)

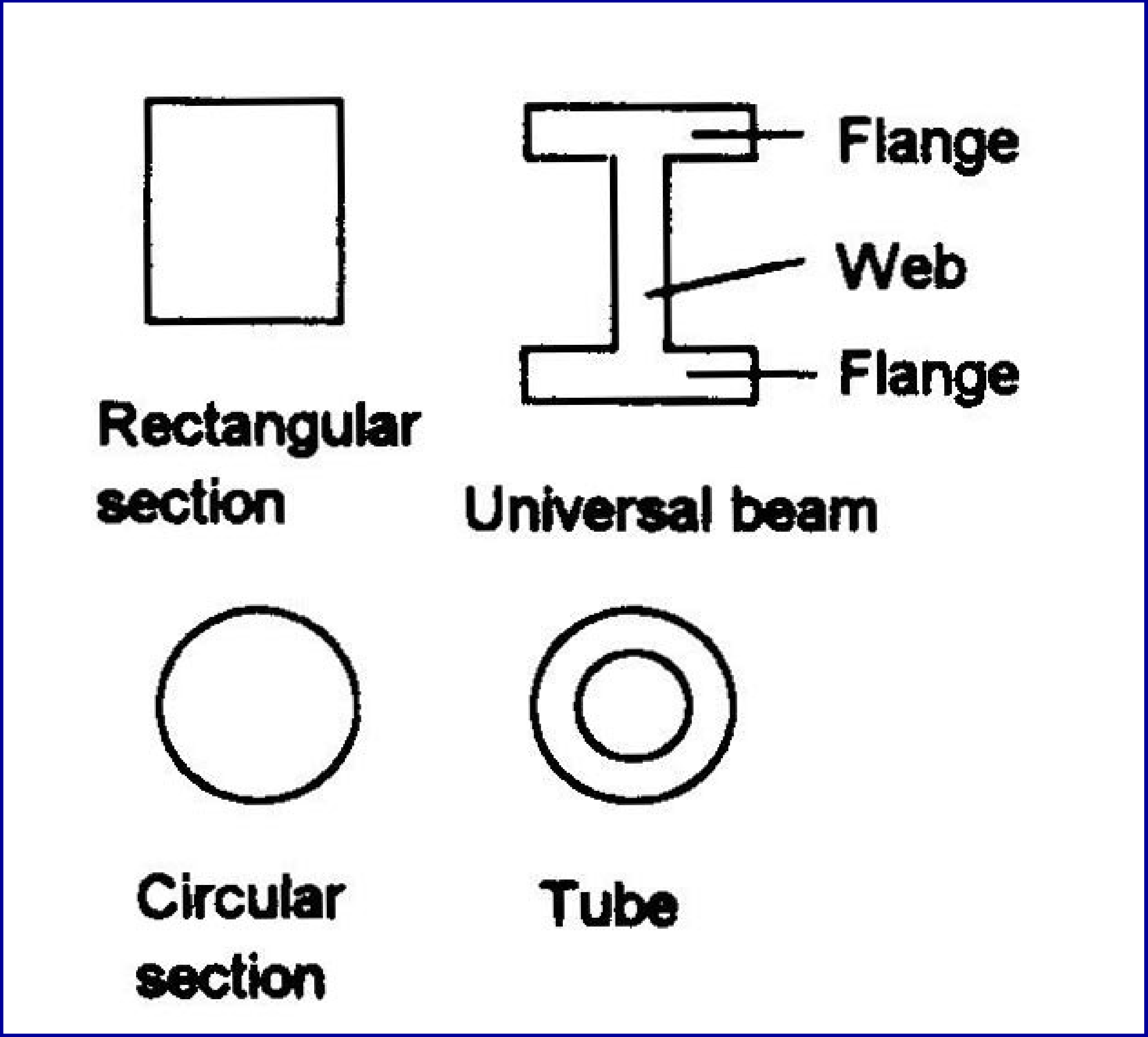


e.g. UDL – 20N/m, 3kN/m

Combined loading

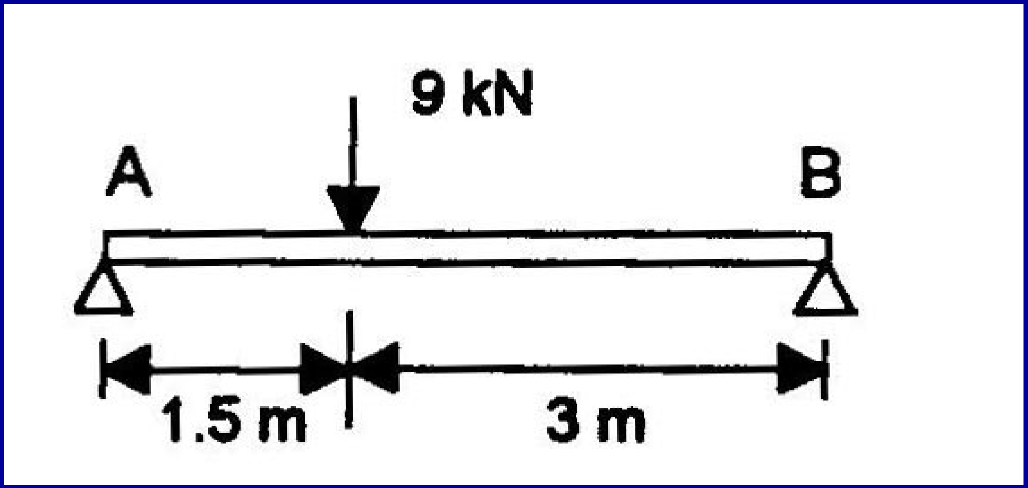


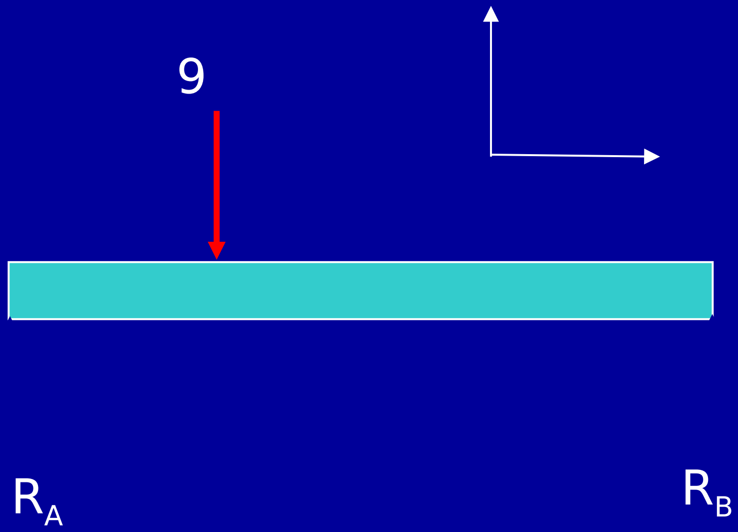
Beams can have a range of different forms of section.



Example

Determine the reactions of a beam of length 4.5 m which is supported at its ends and subject to a point load of 9 kN a distance of 1.5 m from the left-hand end. Neglect the weight of the beam.





The reactions at the supports can be found by taking moments about LHS:

$$\Sigma M_A = 0 = R_B \times 4.5 - 9 \times 1.5$$

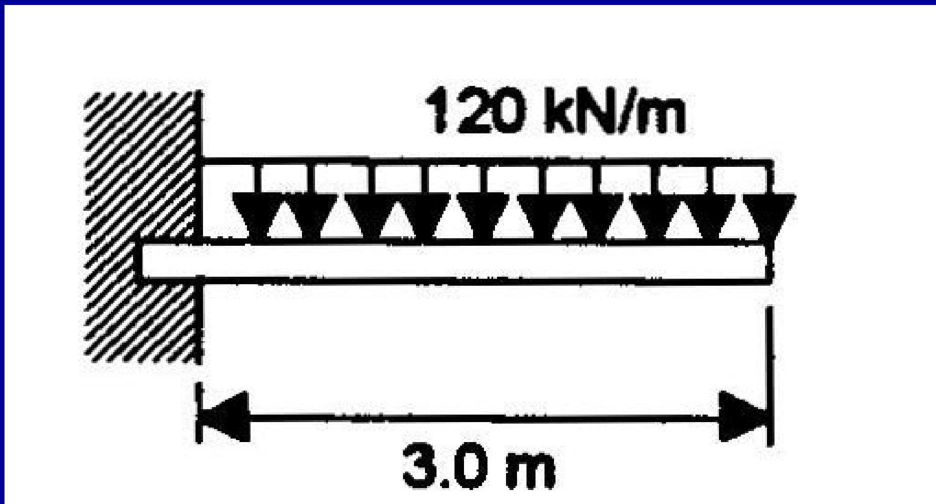
$$\therefore R_B = (9 \times 1.5)/4.5 = \underline{3kN}$$

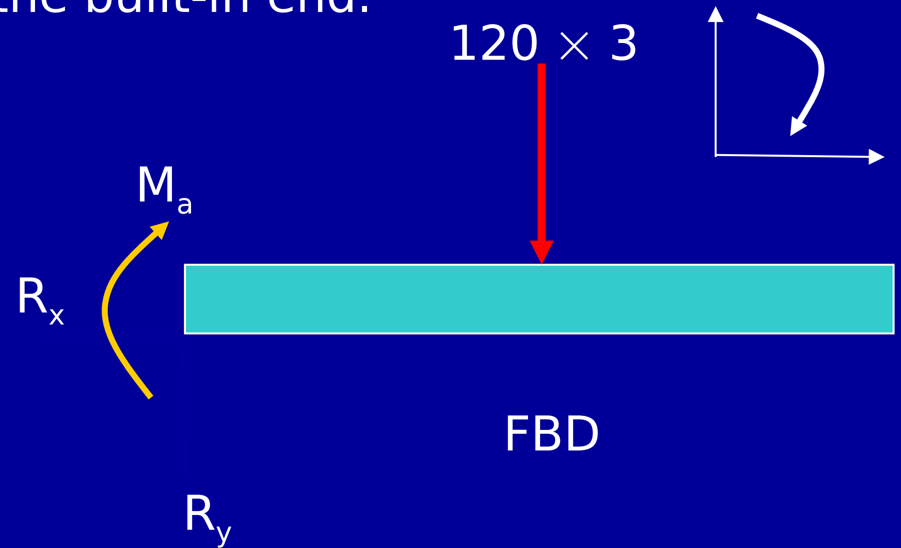
$$\Sigma F_y = 0 = R_B + R_A - 9$$

$$\therefore R_A = \underline{6kN}$$

Example

A uniform cantilever of length 3.0m has a uniform weight per metre of 120kN. Determine the reactions at the built-in end.





First <u>convert</u> the <u>UDL</u> into a <u>point load</u>:

$120 \times 3 = 360$kN acting at the mid point i.e. 1.5m from either end.

By inspection $R_x = 0$ as NO forces acting in the x-axis.

$\Sigma M_A = 0 = 360 \times 1.5 + M_a$

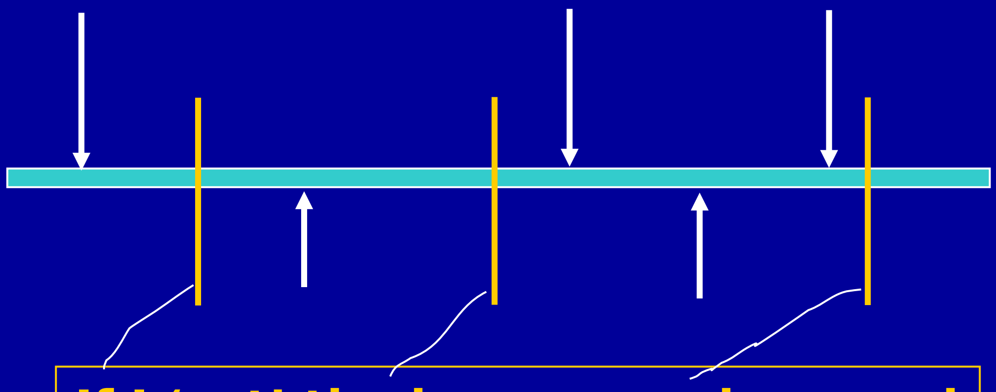
$\therefore M_a = -360 \times 1.5 \boxed{=} -\underline{540}$ $\underline{\text{kNm}}$

Therefore, direction of M_a is

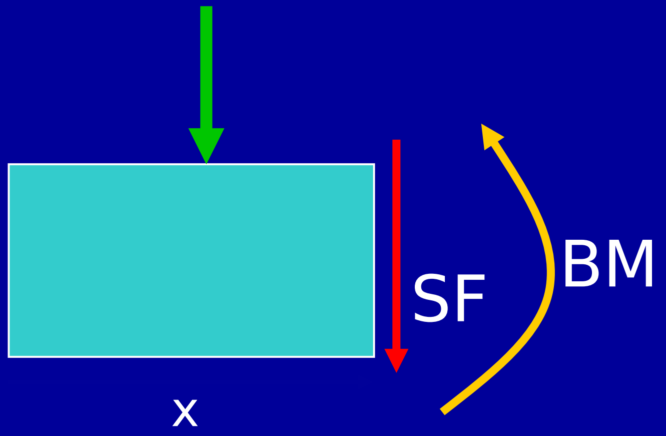
$\Sigma F_y = 0 = R_y - 360$

$\therefore R_y = \underline{360 \text{ kN}}$

Shear Force & Bending Moments.



If I 'cut' the beam anywhere and examine what is happening, I shall see the following:



SHEAR FORCE

- The algebric sum of the vertical forces on either side of the section of a loaded beam is called Shearing Force

Bending Moment

- The algebric sum of the moments of the forces on either side of the section of a loaded beam is called Bending Moment.

Example.

A uniform cantilever of length 3.0m has a uniform weight per metre of 120kN. Determine the shear force and bending moment at distances of

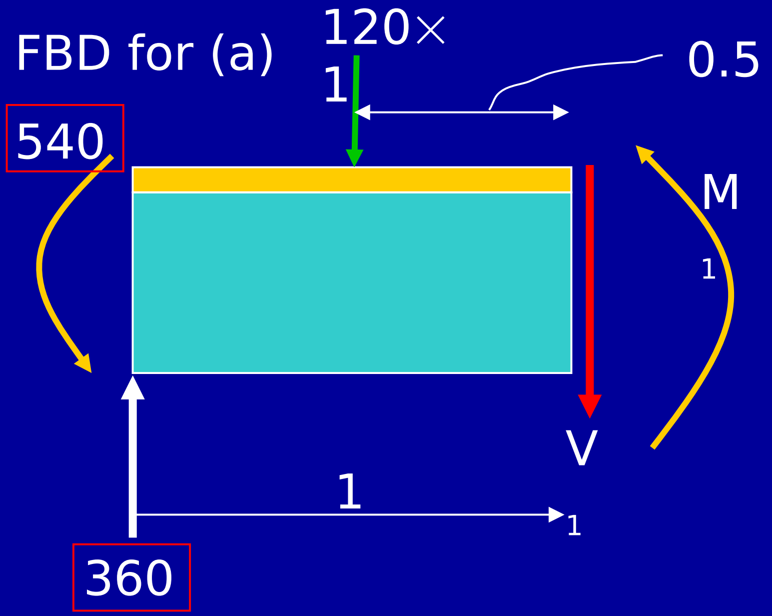
(a)1.0m.

(b)2.0 m

from the built-in end if no other loads are carried by the beam.

Reactions have already been determined – slide 10

FBD for (a)



You will notice that in tackling SF & BM problems you set up $\Sigma F_y = 0$ to determine the SF (i.e. V) AND $\Sigma M = 0$ to determine the BM (i.e. M)

To determine the shear force at 1m from the LHS:

$$\Sigma F_y = 0 = 360 - 120 - V_1$$

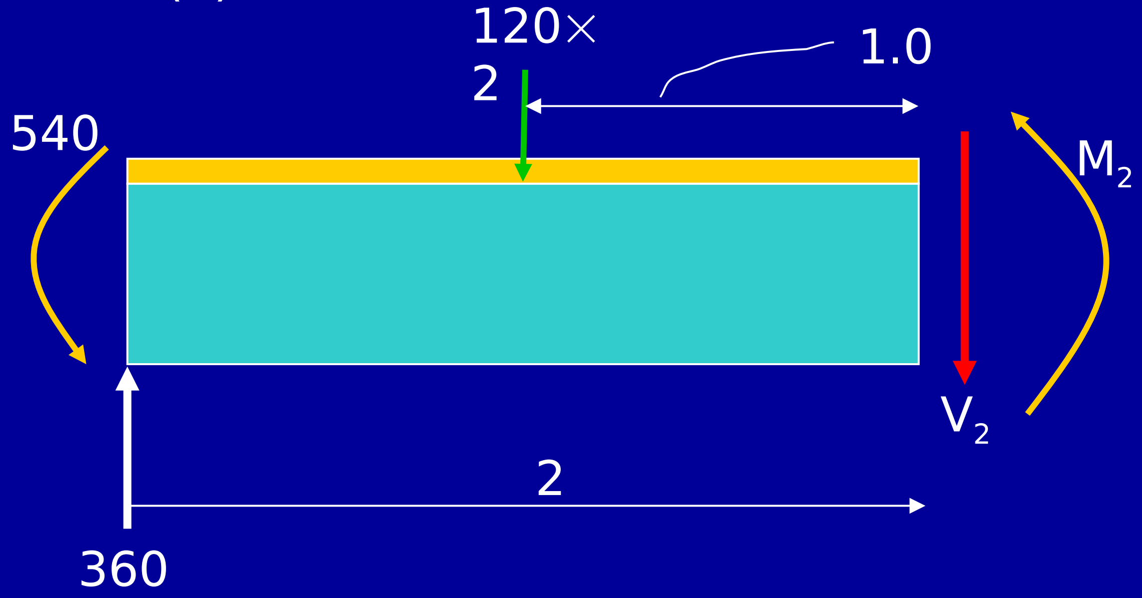
$$\therefore V_1 = \underline{240 \text{ kN}}$$

To determine the bending moment at 1m from the LHS:

$$\Sigma M_{\text{at 1m}} = 0 = -540 + (360 \times 1) - (120 \times 0.5) - M_1$$

$$\therefore M_1 = -540 + 360 - 60 = -\underline{240 \text{ kNm}}$$

FBD for (b)



To determine the shear force at 2m from the LHS:

$\Sigma F_y = 0 = 360 – 240 – V_2$

$\therefore V_2 =$ <u>120 kN</u>

To determine the bending moment at 2m from the LHS:

$\Sigma M_{at\ 2m} = 0 = -540 + (360 \times 2) - (240 \times 1) – M_2$

$\therefore M_2 = - 540 + 720 - 240 =$ <u>- 60 kNm</u>

- If we wanted to determine the SF and BM at any point along the beam, would need to go through this process each time.
- You will agree this tedious.
- We need an alternative method to enable us to determine SF and BM at any point along the beam.

Shear force and bending moment diagrams.

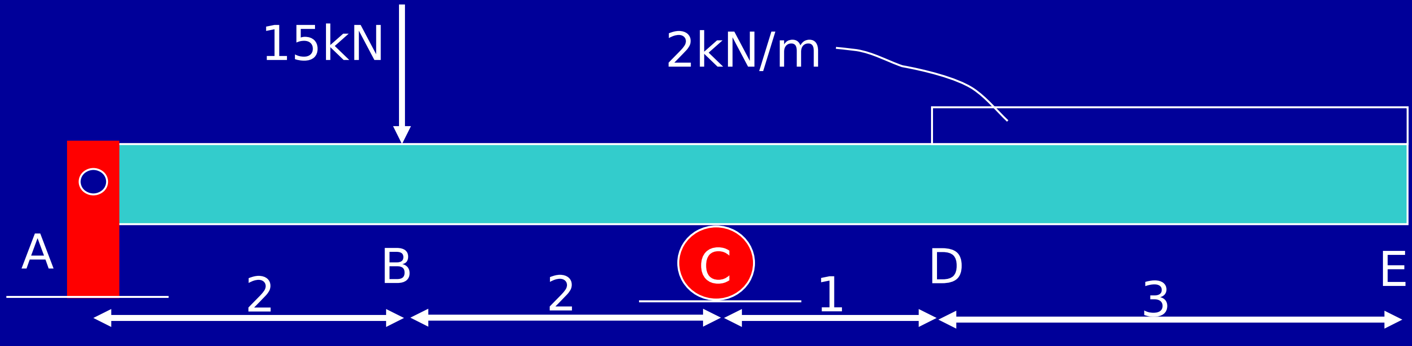
- Shear force diagrams and bending moment diagrams are graphs used to show the variations of the shear forces and bending moments along the length of a beam.

A steel beam 8m long is pin-jointed at the left-hand end and simply supported 4m from the right-hand end. The beam is loaded as shown. For the beam:

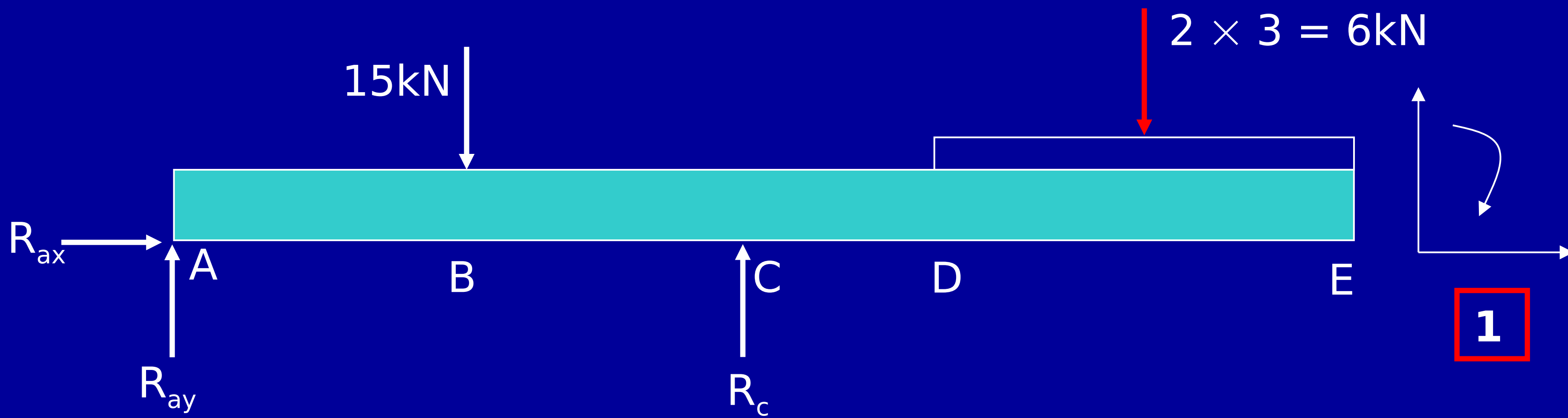
(a) Determine the reactions at A and C.

(b) Derive equations for the shear force and bending moment as a function of distance ‘x’ (horizontal displacement from the left-hand end).

(c) Draw the shear force and bending moment diagrams for the beam, and determine the position of counteraflexure (should one exist).



(a) Draw the FBD.



By inspection $R_{ax} = 0$

Have TWO unknowns: R_{ay} and R_c. Therefore, require TWO equations:

$\Sigma F_y = 0$ & $\Sigma M = 0$

$\Sigma F_y = 0 = R_{ay} - 15 + R_c - 6$

$\therefore R_{ay} = 21 - R_c$ (1)

$\Sigma M_A = 0 = (15 \times 2) - (R_c \times 4) + (6 \times 6.5)$

$30 - 4R_c + 39 = 0$

$\therefore R_c = 69/4 =$ 17.25kN

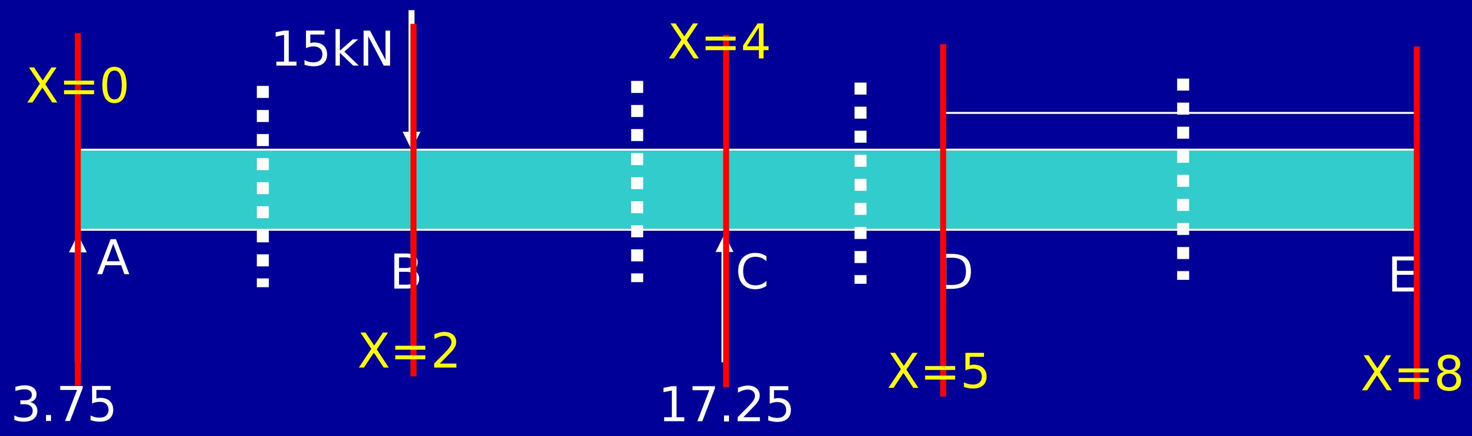
2

Sub for R_c into equation (1)

$\therefore R_{ay} = 21 - 17.25 = \underline{3.75\ \text{kN}}$

(b) To derive the equations, split the beam into a number of spans.

Start from the LHS and whenever you come across a force OR bending moment, you get a span for which an equation has to be derived.



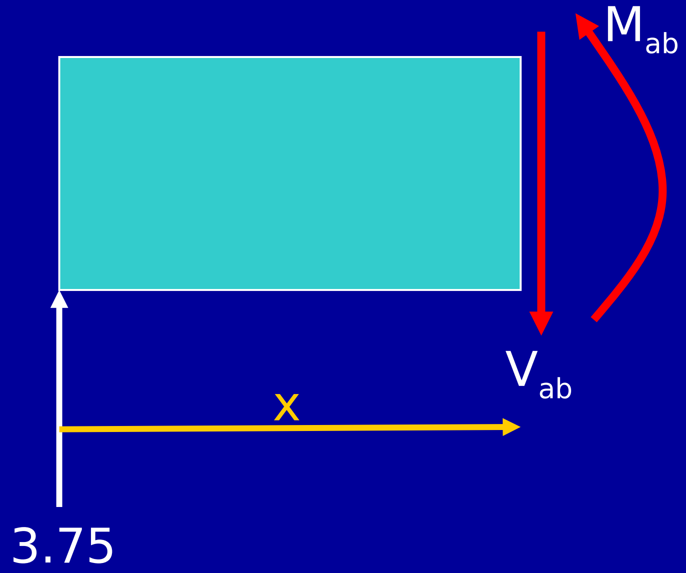
Span AB: $0 \le x \le 2$

Span BC: $2 \le x \le 4$

Span CD: $4 \le x \le 5$

Span DE: $5 \le x \le 8$

Span AB: $0 \le x \le 2$



$\Sigma F_y = 0 = 3.75 - V_{ab}$

$\therefore V_{ab} = \underline{3.75 \text{ kN}}$

$\Sigma M_{AB} = 0 = 3.75x - M_{ab}$

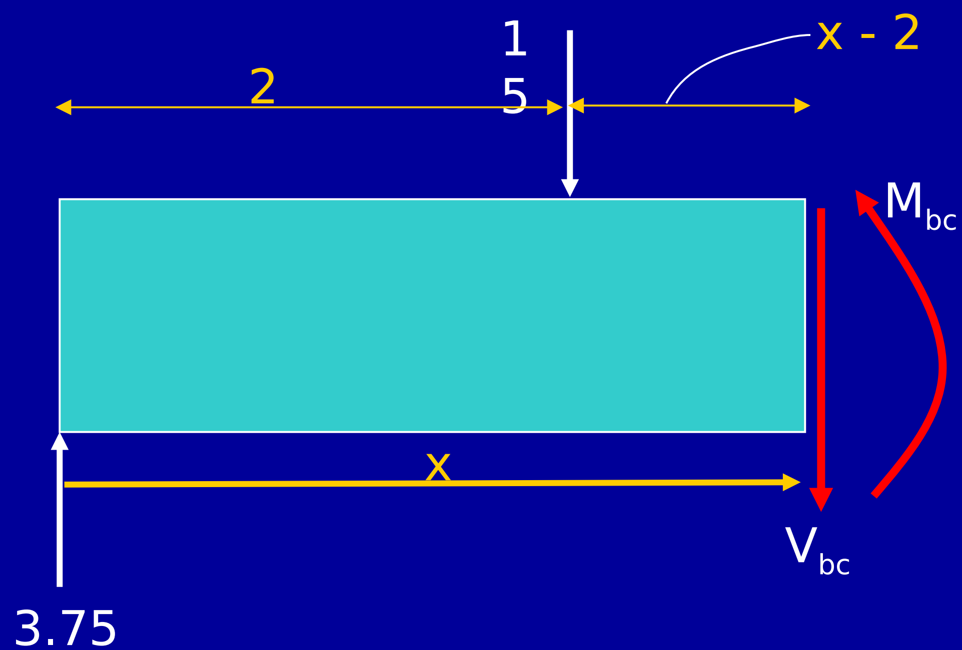
$\therefore M_{ab} = \underline{3.75x \text{ kNm}}$

A check here is useful.

$M = 3.75x$

$\delta M/\delta x = 3.75 = V_{ab}$ ✓

Span BC: $2 \leq x \leq 4$



$\Sigma F_y = 0 = 3.75 - 15 - V_{cb}$

$\therefore V_{bc} = \underline{-11.25 \text{ kN}}$

$\Sigma M_{BC} = 0 = 3.75x - 15(x - 2) - M_{bc}$

$0 = 3.75x - 15x + 30 - M_{bc}$

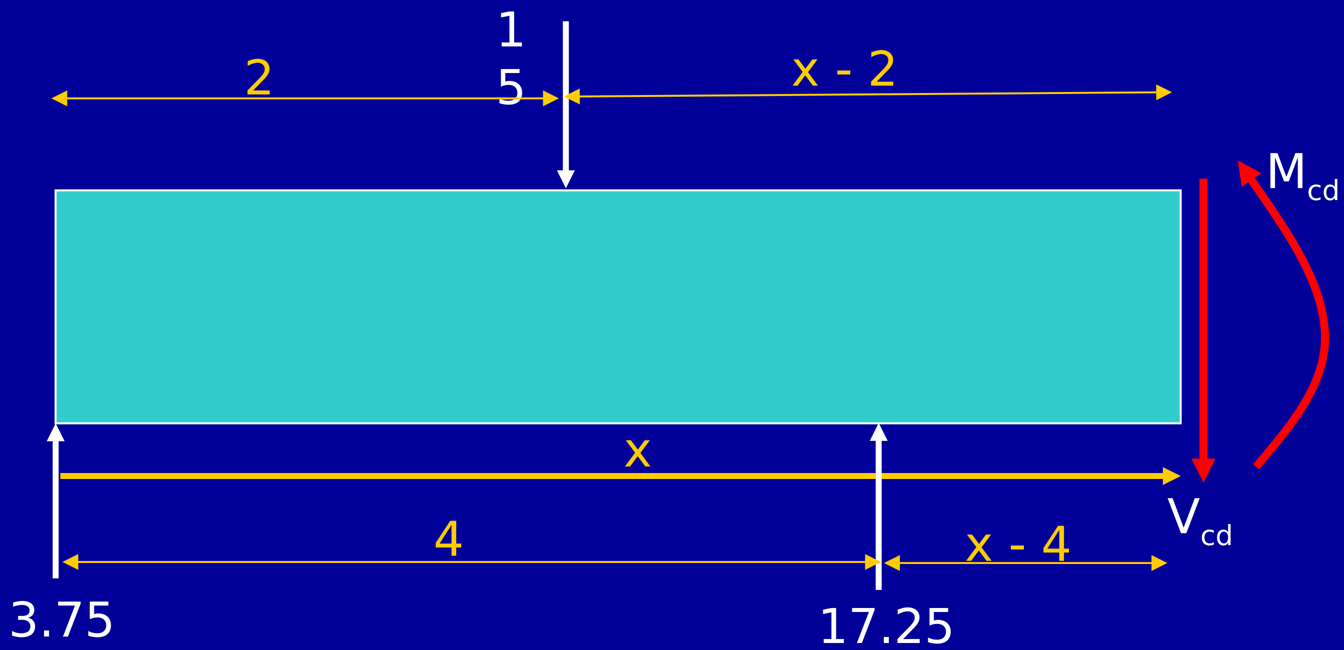
$\therefore M_{bc} = \underline{-11.25x + 30 \text{ kNm}}$

CHECK:

$M = -11.25x + 30$

$\delta M/\delta x = -11.25 = V_{bc}$ ✓

Span CD: $4 \leq x \leq 5$



$\Sigma F_y = 0 = 3.75 – 15 + 17.25 - V_{cd}$

$\therefore V_{cd} = \underline{6 \text{ kN}}$

$\Sigma M_{CD} = 0 = 3.75x – 15(x – 2) + 17.25(x – 4) - M_{cd}$

$0 = 3.75x – 15x + 30 + 17.25x – 69 - M_{cd}$

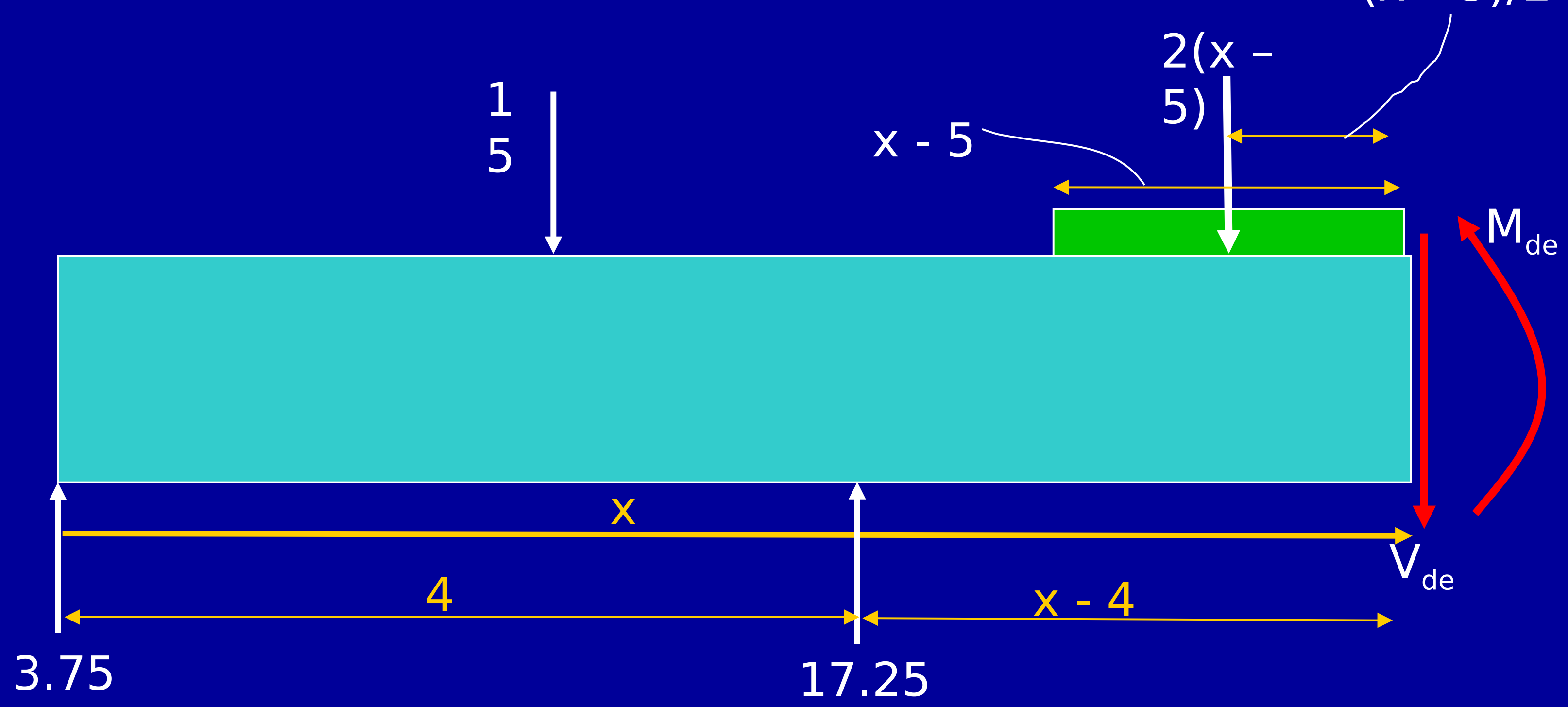
$\therefore M_{cd} = \underline{6x - 39 \text{ kNm}}$

CHECK:

$M = 6x - 39$

$\delta M/\delta x = 6 = V_{cd}$ ✓

Span DE: $5 \le x \le 8$



$$\Sigma F_y = 0 = 3.75 - 15 + 17.25 - (2x - 10) - V_{de}$$

$$0 = 3.75 - 15 + 17.25 - 2x + 10 - V_{de}$$

$$\therefore V_{de} = \underline{-2x + 16 \text{ kN}}$$

$$\Sigma M_{DE} = 0 = 3.75x - 15(x - 2) + 17.5(x - 4) - (2x - 10)(0.5x - 2.5) - M_{de}$$

$$0 = 3.75x - 15x + 30 + 17.5x - 69 - x^2 + 10x - 25 - M_{de}$$

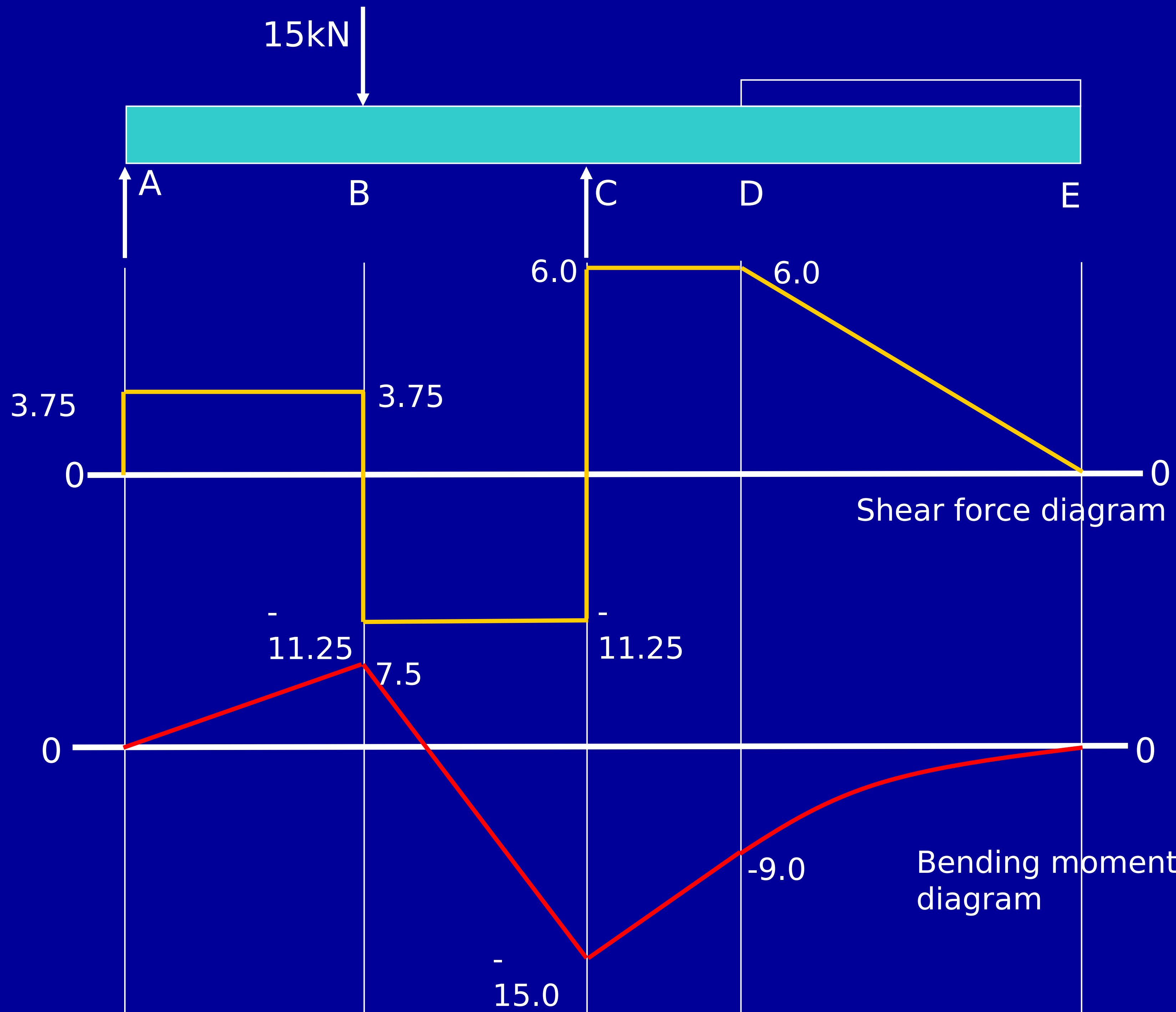
$$\therefore M_{de} = \underline{-x^2 + 16 - 64 \text{ kNm}}$$

CHECK:

$$M = -x^2 + 16x - 64$$

$$\delta M/\delta x = -2x + 16 = V_{de}$$ ✓

(c) To plot the shear force and bending moment diagrams is now a straight forward process of putting in the limits positions of ‘x’ in each of the equations and plotting the values on a graph.


15kN
A
B
C
D
E
3.75
3.75
6.0
6.0
0
0
-
11.25
-
11.25
Shear force diagram
7.5
0
0
-9.0
-
15.0
Bending moment
diagram

Point of counterflexure is a point where the bending moment graph crosses the axis.

There are situations where there is NO point of counterflexure, i.e. the bending moment graph DOES NOT cross the axis.

At the point of counterflexure, the bending moment is EQUAL TO ZERO.

In this question we do have point of counterflexure.

This occurs over span BC, .i.e. when $2 \le x \le 4$

$\therefore M_{bc} = -11.25x + 30 = 0$

$\therefore x = 30/11.25 = 2.67m$

$\therefore$The point of counterflexure is when x = <u>2.67m</u>